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Active control of nonlinear piezoelectric circular shallow spherical shells

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Abstract

Nonlinear electromechanics and active control of a piezoelectric laminated circular spherical shallow shell are quantitatively investigated in this paper. It is assumed that the piezoelectric layers are uniformly distributed on the top and the bottom surfaces of the shell and the thickness of piezoelectric layers is much thinner than that of the shell. The governing equations for the nonlinear dynamics of active control of the circular spherical shallow shell with piezoelectric actuator are formulated and a semi-analytical method is employed to solve the nonlinear governing equations. The numerical results show that the configuration of nonlinear deformation and the natural frequency of the shell structures can be actively controlled by means of high control voltages across the piezoelectric layers and the jumping phenomenon may occur for the case of geometrical parameter $\gamma = [3(1 - \mu^2)]^{1/2} f/h \geq 1$. In addition the effect of large amplitude on the vibrating frequency is discussed by the Galerkin method and the KBM perturbation. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

The importance of piezoelectric materials has been considerably intensified by researchers and structural engineers in recent years because of their electromechanical property. Research and experiments show that piezoelectric materials can be used as actuators/sensors to control structural configurations and to suppress some undesired vibrations in, e.g., space structures, mirrors of telescopes, antennas, robots, rotor systems and high-precision systems, etc. Most of these structures, especially space structures, are required to be lightweight. Thus, the structures are often flexible and nonlinear deformations under external static and dynamic excitations. In this case, nonlinear effect should be considered in the development for structural controls.

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Linear theory of piezoelectric structures is established by using laminated structure theory on the basis of mutual effects between piezoelectricity and mechanics of the structures (Tzou and Anderson, 1992; Hubbard and Burke, 1992; Tzou, 1993, etc.). The numerical method such as the finite element method is proposed to quantitatively analyze the dynamic control of piezoelectric structures (Tzou and Tsing, 1991; Tzou and Zhong, 1993; Hwang and Park, 1993). Since it is much more difficult to get a set of solutions for a nonlinear system than for a linear system, the theoretical researches for nonlinear piezoelectric structures are mainly focused on the development of theoretical models but few quantitative results for them are found in the literature. Librescu (1987) proposed a refined geometrically nonlinear theory of anisotropic laminated shells. Sreeram et al. (1993) studied the nonlinear hysteresis modeling of piezoelectric actuators. Lalande et al. (1993) investigated the nonlinear deformation of a piezoelectric Moonie actuator based on a simplified beam theory. Pai et al. (1993) analyzed a composite plate laminated with piezoelectric layers. Yu (1993) reviewed the recent studies of linear and nonlinear theories of elastic and piezoelectric plates. Zhou et al. (1994) studied the nonlinear behavior of a vibrating diaphragm with nonlinear deformation quantitatively. Tzou and Zhou (1995) quantitatively investigated the active control of nonlinear piezoelectric circular plates.

In this paper, the active control for static and dynamic behavior of piezoelectric laminated circular spherical shells with geometrically nonlinear deformation is investigated. The nonlinear governing equations of the piezoelectric shells with the von Karman type nonlinear deformation are formulated. The numerical results for nonlinear static configuration and the natural frequency of small vibration in the vicinity of the configuration of the shells imposed by the piezoelectric actuators via high input voltage are obtained by a semi-analytical method. It is shown that both the configuration and the natural frequency can be controlled by the applied voltage across the piezoelectric layers and the snapping phenomenon may occur for a large geometrical parameter of the shell. Finally, the effect of large amplitudes of nonlinear free vibration on the vibrating frequency is discussed quantitatively, from which, the extent of stability for the nonlinear free vibration is displayed.

2. Nonlinear mechanics of a piezoelectric shell

For simplicity, here we will concentrate our attention on the axisymmetric deformation of a circular spherical shallow shell subjected to piezoelectric actuators. It is assumed that this shell is laminated with two piezoelectric layers. The piezoelectric layers are uniformly distributed on the top and the bottom of the shell. The same effect of the piezoelectric layers on the deformation are considered in both radial and circumferential directions. Further, the thickness of piezoelectric layers is much thinner than that of the elastic shell. From laminated shell theory (Tzou, 1993), we can write the mechanical equations of the shell with geometrical nonlinearity as follows.

2.1. Equations of motion

$$N_r - N_\theta + r \frac{\partial N_r}{\partial r} = 0 \quad (1)$$

$$M_r + r \frac{\partial M_r}{\partial r} - M_\theta + Q_r = 0 \quad (2)$$

$$\frac{\partial}{\partial r}(rQ_r) + \frac{\partial}{\partial r} \left[rN_r \left(\frac{dz}{dr} + \frac{\partial w}{\partial r} \right) \right] - \rho hr \frac{\partial^2 w}{\partial t^2} = 0 \quad (3)$$

in which

$$N_r = N_r^m - N_r^e, \quad N_\theta = N_\theta^m - N_\theta^e \quad (4a,b)$$

$$M_r = M_r^m - M_r^e, \quad M_\theta = M_\theta^m - M_\theta^e \quad (5a,b)$$

2.2. Geometrical relations

After the geometrical relationship of von Karman's type for the deformation of the thin shell is considered, the geometrical equations describing the nonlinear deformation can be written as

$$\kappa_r = -\frac{\partial^2 w}{\partial r^2}, \quad \kappa_\theta = -\frac{1}{r} \frac{\partial w}{\partial r} \quad (6a,b)$$

$$\varepsilon_r = \frac{\partial u}{\partial r} + \frac{dz}{dr} \frac{\partial w}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2, \quad \varepsilon_\theta = \frac{u}{r} \quad (7a,b)$$

2.3. Constitutive equations

Here, the linear mechanical and electrical materials of the piezoelectric spherical shell are given as follows

$$M_r^m = D(\kappa_r + \mu\kappa_\theta), \quad M_\theta^m = D(\kappa_\theta + \mu\kappa_r) \quad (8a,b)$$

$$\varepsilon_r = \frac{1}{Yh}(N_r^m + \mu N_\theta^m), \quad \varepsilon_\theta = \frac{1}{Yh}(N_\theta^m + \mu N_r^m) \quad (9a,b)$$

and

$$N_r^e = N_\theta^e = e_{31}(\phi_3^u + \phi_3^d)/2 \quad (10)$$

$$M_r^e = M_\theta^e = e_{31}(h + h_p)(\phi_3^u - \phi_3^d)/2 \quad (11)$$

Here, the superscripts 'm' and 'e' represent the mechanical and electric parts of the quantities, respectively; N_r and N_θ are membrane forces in the r - and θ -directions, respectively; M_r and M_θ are bending moments in the r - and θ -directions, respectively; κ_r and κ_θ represent the increment of curvatures of the deformed shell from its undeformed state in the r - and θ -directions, respectively; u and w are the radial displacement and transverse deflections of the shell, respectively; Y and μ are Young's modulus and Poisson's ratio, respectively; h and h_p denote thicknesses of the shell and the piezoelectric layers, respectively; $D(= Yh^3/[12(1 - \mu^2)])$ is the bending stiffness; ϕ_3^u and ϕ_3^d are control voltages across the top and the bottom piezoelectric actuator layers, respectively; ρ is the mass density of the shell; t is the time variable. The piezoelectric layers are assumed to be effective in only the radial and circumferential directions and to have identical piezoelectric constants. This is, $e_{31} = e_{32}$. For the circular shallow spherical shell shown in Fig. 1, the undeformed form of the shell can be formulated by

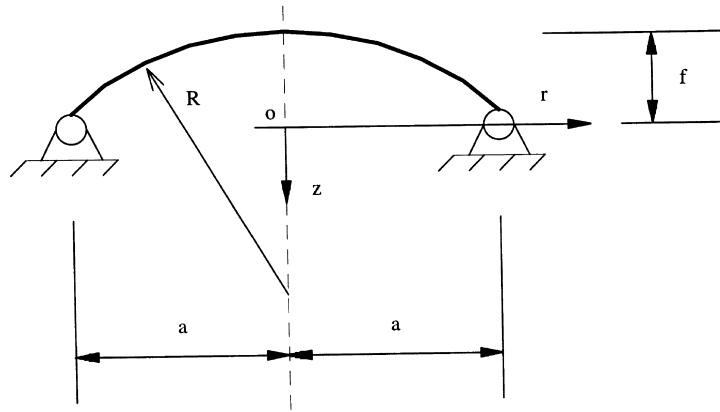


Fig. 1. A piezoelectric laminated nonlinear shallow spherical shell.

$$z(r) = -f\left(1 - \frac{r^2}{a^2}\right) \tag{12}$$

in which f is the height of the shell and a is the radius of the circular edge of the shell (see Fig. 1).

2.4. Governing equations

Eliminating N_θ , M_r , M_θ , Q_r , Q_θ , κ_r , κ_θ , ϵ_r , ϵ_θ and u in eqns (1)–(11) and considering the condition of applicability for shallow spherical shells (Zhou, 1989), one can obtain the following nonlinear differential equations with two independent unknowns w and N_r^m :

$$DL_r[w(r, t)] = -\rho h \frac{\partial^2 w}{\partial t^2} + \frac{1}{r} \frac{\partial}{\partial r} \left[r N_r^m \left(\frac{2f}{a^2} r + \frac{\partial w}{\partial r} \right) \right] - \frac{1}{r} \frac{\partial}{\partial r} \left[r N_r^e \left(\frac{2f}{a^2} r + \frac{\partial w}{\partial r} \right) \right] - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial M_r^e}{\partial r} \right) \tag{13}$$

$$r \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r^2 N_r^m) \right] = -Yh \left[\frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2 + \frac{2f}{a^2} r \frac{\partial w}{\partial r} \right] + \frac{\partial}{\partial r} \left(r^2 \frac{\partial N_r^e}{\partial r} \right) - \mu r \frac{\partial N_r^e}{\partial r} \tag{14}$$

where $0 < r < a$ and

$$L_r = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \tag{15}$$

For the case of a stationary simply supported edge, the boundary conditions are

$$r = 0: \quad \frac{\partial w}{\partial r} = 0; \quad N_r^m \text{ finite} \tag{16a,b}$$

$$r = a: \quad w = 0; \quad \frac{\partial}{\partial r} (r N_r^m) - \mu N_r^m = 0; \quad D \left(\frac{\partial^2 w}{\partial r^2} + \frac{\mu}{r} \frac{\partial w}{\partial r} \right) + M_r^e = 0 \tag{17a,b,c}$$

In the following discussion, we will focus our attention on the case of the full bonded piezoelectric layers with constant voltage $\phi_3^u = -\phi_3^d$. For this case, we have

$$N_r^e = N_\theta^e = 0 \tag{18a,b}$$

$$M_r^e = M_\theta^e = -e_{31}(h + h_p)\phi_3^u \tag{19a,b}$$

3. Decomposition of response

In order to find the solution of the nonlinear piezoelectric shell, we take a semi-analytical and semi-numerical approach in which the response is decomposed into static and dynamic parts. Let $w_0(r)$ and $N_{r0}^m(r)$ be the solutions of the nonlinear static state of the shell, that is, they are governed by the nonlinear equations

$$DL_r[w_0(r)] = \frac{1}{r} \frac{d}{dr} \left[r N_{r0}^m \left(\frac{2f}{a^2} r + \frac{dw_0}{dr} \right) \right] \quad 0 < r < a \tag{20}$$

$$r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r^2 N_{r0}^m(r)) \right] = -Yh \left[\frac{1}{2} \left(\frac{dw_0}{dr} \right)^2 + \frac{2f}{a^2} r \frac{dw_0}{dr} \right] \quad 0 < r < a \tag{21}$$

with the boundary conditions

$$r = 0: \quad \frac{dw_0}{dr} = 0; \quad N_{r0}^m \text{ finite} \tag{22a,b}$$

$$r = a: \quad w_0 = 0; \quad \frac{d}{dr} (r N_{r0}^m) - \mu N_{r0}^m = 0; \quad D \left(\frac{d^2 w_0}{dr^2} + \frac{\mu}{r} \frac{dw_0}{dr} \right) + M_r^e = 0 \tag{23a,b,c}$$

Decomposing the response of $w(r, t)$ and $N_r^m(r, t)$ into the static part, $w_0(r)$ and $N_{r0}^m(r)$ and the dynamic part, $w_t(r, t)$ and $N_{rt}^m(r, t)$ which are measured in the vicinity of the static configuration of the piezoelectric shell, we can write

$$w(r, t) = w_0(r) + w_t(r, t) \tag{24}$$

$$N_r^m(r, t) = N_{r0}^m(r) + N_{rt}^m(r, t) \tag{25}$$

Substituting eqns (24) and (25) into the governing eqns (13)–(17), then, subtracting the static eqns (20)–(23) from them, one can obtain the governing equations for the dynamic part of the response, i.e.,

$$DL_r[w_t(r, t)] = -\rho h \frac{\partial^2 w_t}{\partial t^2} + \frac{1}{r} \frac{\partial}{\partial r} \left[r N_{rt}^m \left(\frac{2f}{a^2} r + \frac{dw_0}{dr} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left(r N_{r0}^m \frac{\partial w_t}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r N_{rt}^m \frac{\partial w_t}{\partial r} \right) \quad 0 < r < a \tag{26}$$

$$r \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r^2 N_{rt}^m) \right] = -Yh \left[\left(\frac{dw_0}{dr} \right) \left(\frac{\partial w_t}{\partial r} \right) + \frac{2f}{a^2} r \frac{\partial w_t}{\partial r} + \frac{1}{2} \left(\frac{\partial w_t}{\partial r} \right)^2 \right] \quad 0 < r < a \tag{27}$$

with boundary conditions

$$r = 0: \quad \frac{\partial w_t}{\partial r} = 0; \quad N_{rt}^m \text{ finite} \quad (28a,b)$$

$$r = a: \quad w_t = 0; \quad \frac{\partial}{\partial r}(rN_{rt}^m) - \mu N_{rt}^m = 0; \quad D\left(\frac{\partial^2 w_t}{\partial r^2} + \frac{\mu}{r} \frac{\partial w_t}{\partial r}\right) = 0 \quad (29a,b,c)$$

4. Solutions of eqns (20)–(23)

After the following dimensionless quantities

$$y = \left(\frac{r}{a}\right)^2, \quad W_0 = [3(1 - \mu^2)]^{1/2} \frac{w_0}{h}, \quad \gamma = [3(1 - \mu^2)]^{1/2} \frac{f}{h}, \quad \Phi_0 = y \frac{dW_0}{dy},$$

$$S_0^m(y) = 3(1 - \mu^2) \frac{a^2 N_{r0}^m}{Yh^3} y, \quad T_0^m(y) = 6(1 - \mu^2) [3(1 - \mu^2)]^{1/2} \frac{a^2 M_{r0}^m}{Yh^4},$$

$$T_0^c = 6(1 - \mu^2) [3(1 - \mu^2)]^{1/2} \frac{a^2 M_{r0}^c}{Yh^4} \quad (30)$$

are introduced, the governing eqns (20)–(23) of the nonlinear static deformation can be reduced to (Zheng and Zhou, 1990)

$$y^2 \frac{d^2 \Phi_0(y)}{dy^2} = \Phi_0(y) S_0^m(y) + \gamma y S_0^m(y) \quad 0 < y < 1 \quad (31)$$

$$y^2 \frac{d^2 S_0^m(y)}{dy^2} = -\frac{1}{2} \Phi_0(y) [\Phi_0(y) + 2\gamma y] \quad 0 < y < 1 \quad (32)$$

with the boundary conditions

$$y = 0: \quad \Phi_0(y) = 0, \quad S_0^m(y) = 0 \quad (33a,b)$$

$$y = 1: \quad (1 + \mu) S_0^m(y) - 2 \frac{dS_0^m(y)}{dy} = 0, \quad (1 - \mu) \Phi_0(y) - 2 \frac{d\Phi_0(y)}{dy} + T_0^c = 0 \quad (34a,b)$$

The solutions of $\Phi_0(y)$ and $S_0^m(y)$ to eqns (31)–(34) can be expressed by the series formulas (Zheng, 1990)

$$\Phi_0(y) = \sum_{i=1}^{\infty} A_i y^i, \quad S_0^m(y) = \sum_{i=1}^{\infty} B_i y^i \quad (35a,b)$$

which have satisfied the boundary conditions of eqns (33a,b). Substitution of eqn (35) into eqns (31)–(32) and (34a,b) leads to the recurrence formulas for the unknown coefficients A_i and B_i , that is,

$$B_i = -\frac{1}{2i(i-1)} \left(\sum_{j=1}^{i-1} A_j A_{i-j} + 2\gamma A_{i-1} \right) \quad i = 2, 3, 4, \dots \tag{36}$$

$$A_i = \frac{1}{i(i-1)} \left(\sum_{j=1}^{i-1} A_j B_{i-j} + \gamma B_{i-1} \right) \quad i = 2, 3, 4, \dots \tag{37}$$

and the nonlinear algebraic equations

$$\sum_{i=1}^{\infty} [(1 + \mu - 2i)B_i] = 0 \tag{38}$$

$$\sum_{i=1}^{\infty} [(1 - \mu - 2i)A_i] = -T_0^c \tag{39}$$

for the independent unknown coefficients A_1 and B_1 . By using the Newton–Raphson method to solve the nonlinear algebraic eqns (38) and (39), one can obtain the values of A_1 and B_1 and then A_i and B_i (for $i \geq 2$) from the recurrence formulas of eqns (36) and (37). Thus, the static state of the piezoelectric spherical shallow shell is obtained for given applied voltage $\phi_3^u = -\phi_3^d$.

5. Solutions of eqns (26)–(29)

5.1. Dimensionless equations

We introduce the following dimensionless quantities

$$x = \frac{r}{a}, \quad \bar{w}_t = 2[3(1 - \mu^2)]^{1/2} \frac{w_t}{h}, \quad \bar{w}_0 = 2[3(1 - \mu^2)]^{1/2} \frac{w_0}{h}, \quad \tau = \omega_n t$$

$$\bar{N}_{rt}^m = 12(1 - \mu^2) \frac{a^2 N_{rt}^m}{Yh^3}, \quad \bar{N}_{r0}^m = 12(1 - \mu^2) \frac{a^2 N_{r0}^m}{Yh^3}, \quad \lambda = \frac{\rho h a^4}{D} \omega_n^2 \tag{40}$$

in which ω_n is a natural frequency of the shell. Then, we write eqns (26)–(29) in the following dimensionless form:

$$L_x[\bar{w}_t(x, \tau)] = -\lambda \frac{\partial^2 \bar{w}_t}{\partial \tau^2} + \frac{1}{x} \frac{\partial}{\partial x} \left[x \bar{N}_{rt}^m \left(4\gamma x + \frac{d\bar{w}_0}{dx} \right) \right] + \frac{1}{x} \frac{\partial}{\partial x} \left(x \bar{N}_{r0}^m \frac{\partial \bar{w}_t}{\partial x} \right) + \frac{1}{x} \frac{\partial}{\partial x} \left(x \bar{N}_{rt}^m \frac{\partial \bar{w}_t}{\partial x} \right) \tag{41}$$

$$x \frac{\partial}{\partial x} \left[\frac{1}{x} \frac{\partial}{\partial x} \left(x^2 \bar{N}_{rt}^m(x, \tau) \right) \right] = - \left[\frac{d\bar{w}_0}{dx} \frac{\partial \bar{w}_t}{\partial x} + 4\gamma x \frac{\partial \bar{w}_t}{\partial x} + \frac{1}{2} \left(\frac{\partial \bar{w}_t}{\partial x} \right)^2 \right] \quad 0 < x < 1 \tag{42}$$

$$x = 0: \quad \frac{\partial \bar{w}_t}{\partial x} = 0, \quad \bar{N}_{rt} \quad \text{finite} \tag{43a,b}$$

$$x = 1: \quad \bar{w}_t = 0, \quad \frac{\partial}{\partial x} \left(x \bar{N}_{rt}^m \right) - \mu \bar{N}_{rt}^m = 0, \quad \frac{\partial^2 \bar{w}_t}{\partial x^2} + \frac{\mu}{x} \frac{\partial \bar{w}_t}{\partial x} = 0 \quad (44a,b,c)$$

5.2. Natural frequency—applied voltage relation

For the case of small free vibrations of the piezoelectric shell in the vicinity of the nonlinear static state, the nonlinear terms on the unknowns \bar{w}_t and \bar{N}_{rt}^m in dynamic eqns (41)–(44) are neglected. Let

$$\bar{w}_t(x, \tau) = R_t(x) \sin \tau, \quad \bar{N}_{rt}^m(x, \tau) = S_t(x) \sin \tau \quad (45)$$

Then, the eigenvalue equations for small vibrations can be deduced from

$$L_x[R_t(x)] = \lambda R_t(x) + \frac{1}{x} \frac{d}{dx} \left[x S_t(x) \left(4\gamma x + \frac{d\bar{w}_0(x)}{dx} \right) + x \bar{N}_{r0}^m(x) \frac{dR_t(x)}{dx} \right] \quad (46)$$

$$x \frac{d}{dx} \left[\frac{1}{x} \frac{d}{dx} (x^2 S_t(x)) \right] = - \left[\frac{d\bar{w}_0(x)}{dx} \frac{dR_t(x)}{dx} + 4\gamma x \frac{dR_t(x)}{dx} \right] \quad 0 < x < 1 \quad (47)$$

$$x = 0: \quad \frac{dR_t}{dx} = 0, \quad S_t(x) \text{ finite} \quad (48a,b)$$

$$x = 1: \quad R_t(x) = 0, \quad \frac{d}{dx} [x S_t(x)] - \mu S_t(x) = 0, \quad \frac{d^2 R_t}{dx^2} + \frac{\mu}{x} \frac{dR_t}{dx} = 0 \quad (49a,b,c)$$

According to the static solutions of eqns (35), we have

$$\frac{d\bar{w}_0}{dx} = \sum_{i=1}^{\infty} \bar{A}_i x^{2i-1}, \quad \bar{N}_{r0}^m = \sum_{i=1}^{\infty} \bar{B}_i x^{2i} \quad (50a)$$

where $\bar{A}_i = 4A_i$ and $\bar{B}_{i-1} = 4B_i$. Let

$$R_t(x) = \sum_{i=0}^{\infty} a_i x^{2i}, \quad S_t(x) = \sum_{i=0}^{\infty} b_i x^{2i} \quad (51a,b)$$

in which a_i and b_i are constants to be determined. Substituting eqns (50) and (51) into eqns (46)–(49), one can obtain the recurrence formulas

$$b_i = -\frac{1}{2i(i+1)} \left[4\gamma a_i + \sum_{j=1}^i j a_j \bar{A}_{i-j+1} \right] \quad i = 1, 2, 3, \dots \quad (52)$$

$$a_{i+2} = \left[\lambda a_i + 8(i+1)\gamma b_i + 2(i+1) \sum_{j=1}^{i+1} (\bar{A}_j b_{i-j+1} + 2j a_j \bar{B}_{i-j+1}) \right] / [16(i+1)^2(i+2)^2] \quad i = 0, 1, 2, \dots \quad (53)$$

and the algebraic equations

$$\sum_{i=0}^{\infty} a_i = 0 \tag{54}$$

$$\sum_{i=1}^{\infty} (2i - 1 + \mu)ia_i = 0 \tag{55}$$

$$\sum_{i=0}^{\infty} (2i - 1 - \mu)ib_i = 0 \tag{56}$$

The recurrence formulas of eqns (52) and (53) show that the only independent constants are a_0 , a_1 and b_0 . Once we express other constants a_i and b_i as the functions of the independent constants and the eigenvalue of the problem, eqns (54)–(56) will become a system of algebraic equations with explicit form on eigenvalue λ and eigenvector $[a_0, a_1, b_0]^T$. This is $[K_{ij}(\lambda)]_{3 \times 3}[a_0, a_1, b_0]^T = \mathbf{0}$. For a nonzero solution, i.e., $[a_0, a_1, b_0]^T \neq 0$, we get a condition to determine the eigenvalue λ similar to Zhou et al. (1994).

5.3. Effect of large amplitude on frequency

Let $U^*(x)$ be a permissible function of deflection which satisfies the boundary conditions of eqns (43a) and (44a,c). Assume an approximate solution of eqns (41) and (42) in the form

$$\bar{w}_t(x, \tau) = U^*(x)f(\tau) \tag{57}$$

in which $f(\tau)$ is an unknown function of variable τ . Substituting eqn (57) into eqns (41) and (42) and applying the Galerkin method to the first equation, we obtain

$$\frac{d^2f(\tau)}{d\tau^2} + f(\tau) + \Gamma_1 f^2(\tau) + \Gamma_2 f^3(\tau) = 0 \tag{58}$$

where

$$\Gamma_1 = C_3/C_2, \quad \Gamma_2 = C_4/C_2 \tag{59a,b}$$

$$\lambda = \omega_n^2 = C_2/C_1 \tag{59c}$$

in which

$$C_1 = \int_0^1 [U^*(x)]^2 x \, dx \tag{60a}$$

$$C_2 = \int_0^1 \left\{ L_x[U^*(x)] - \frac{1}{x} \frac{d}{dx} \left[x \bar{N}_{r0} \frac{dU^*}{dx} + x \bar{N}_{rt}^* \left(4\gamma x + \frac{d\bar{w}_0}{dx} \right) \right] \right\} U^*(x) x \, dx \tag{60b}$$

$$C_3 = - \int_0^1 \frac{1}{x} \frac{d}{dx} \left[x \bar{N}_{rt}^* \left(4\gamma x + \frac{d\bar{w}_0}{dx} \right) + x \bar{N}_{rt}^* \frac{dU^*}{dx} \right] U^*(x) x \, dx \tag{60c}$$

$$C_4 = - \int_0^1 \frac{1}{x} \frac{d}{dx} \left[x \bar{N}_{rt}^{*n} \frac{dU^*}{dx} \right] U^*(x) x \, dx \tag{60d}$$

Here,

$$\bar{N}_{rt}^{*l}(x) = \frac{1}{2x^2} \int_0^1 G(x^2, \xi^2) \frac{1}{\xi} \left(4\gamma\xi + \frac{d\bar{w}_0}{d\xi} \right) \frac{dU^*(\xi)}{d\xi} d\xi \tag{61a}$$

$$\bar{N}_{rt}^{*n}(x) = \frac{1}{2x^2} \int_0^1 G(x^2, \xi^2) \frac{1}{\xi} \left[\frac{dU^*(\xi)}{d\xi} \right]^2 d\xi \tag{61b}$$

in which the kernel function $G(x^2, \xi^2)$ is

$$G(x^2, \xi^2) = \begin{cases} \left(\frac{1-\mu}{1+\mu} \xi^2 + 1 \right) x^2 & x \leq \xi \\ \left(\frac{1-\mu}{1+\mu} x^2 + 1 \right) \xi^2 & x > \xi \end{cases} \tag{62}$$

Using the Krylov–Bogoliubov–Mitropolsky (KBM) perturbation method (Nayfeh and Mook, 1979) to solve eqn (58), we obtain the amplitude-frequency relation

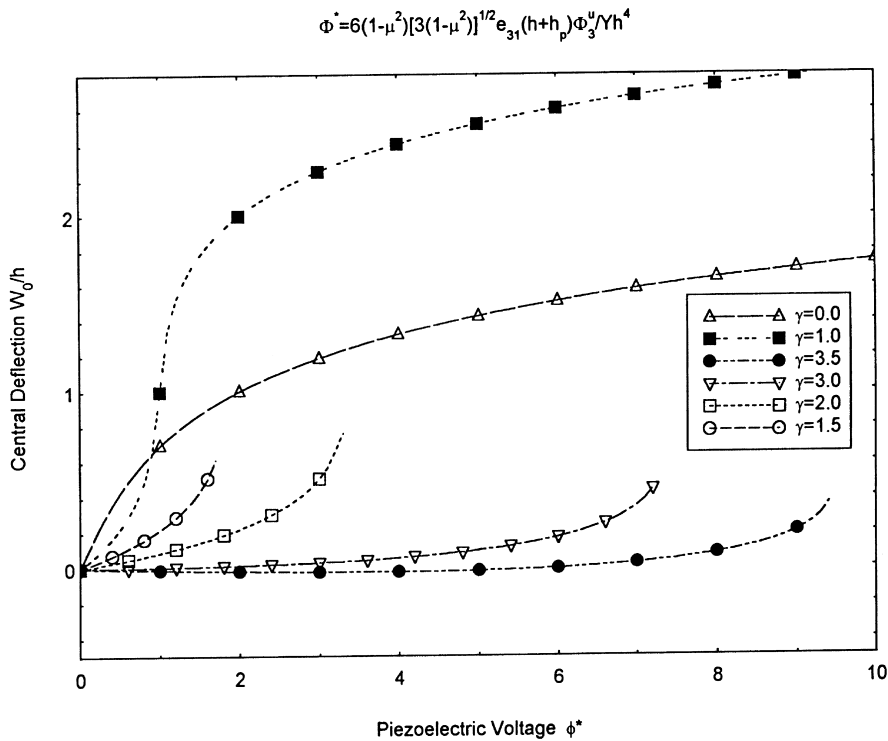


Fig. 2. Characteristic curves of static central deflection $[3(1-\mu^2)]^{1/2}w_0(0)/h$ vs piezoelectric voltage $\phi^* = 6(1-\mu^2)[3(1-\mu^2)]^{1/2}e_{31}(h+h_p)\phi_3^u/Yh^4$. $\mu = 0.3$.

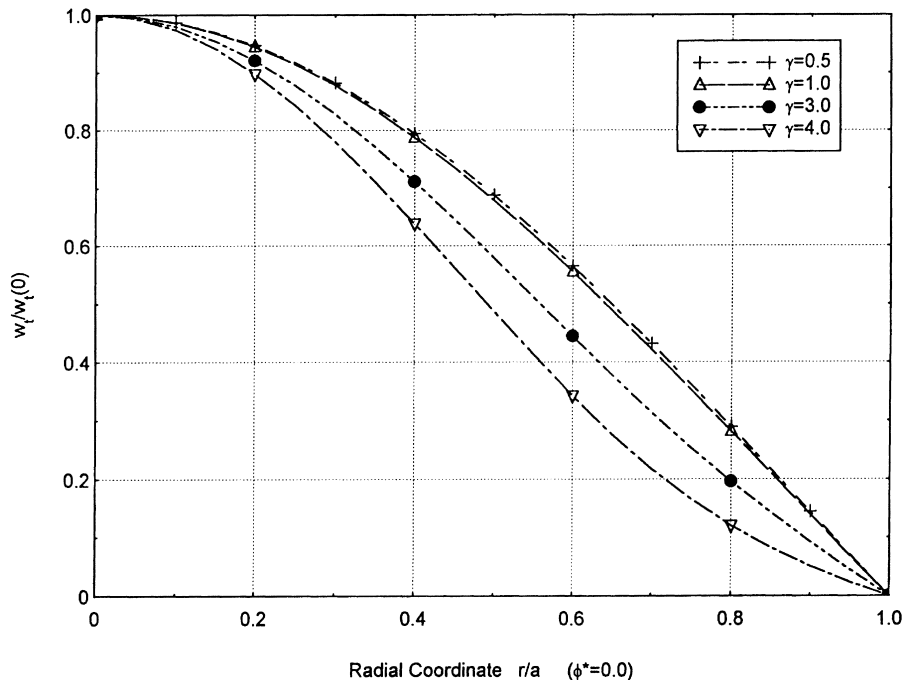


Fig. 3. The first mode shape of deflection of the piezoelectric shallow spherical shells for the small free vibration in the vicinity of configuration of nonlinear static deformation.

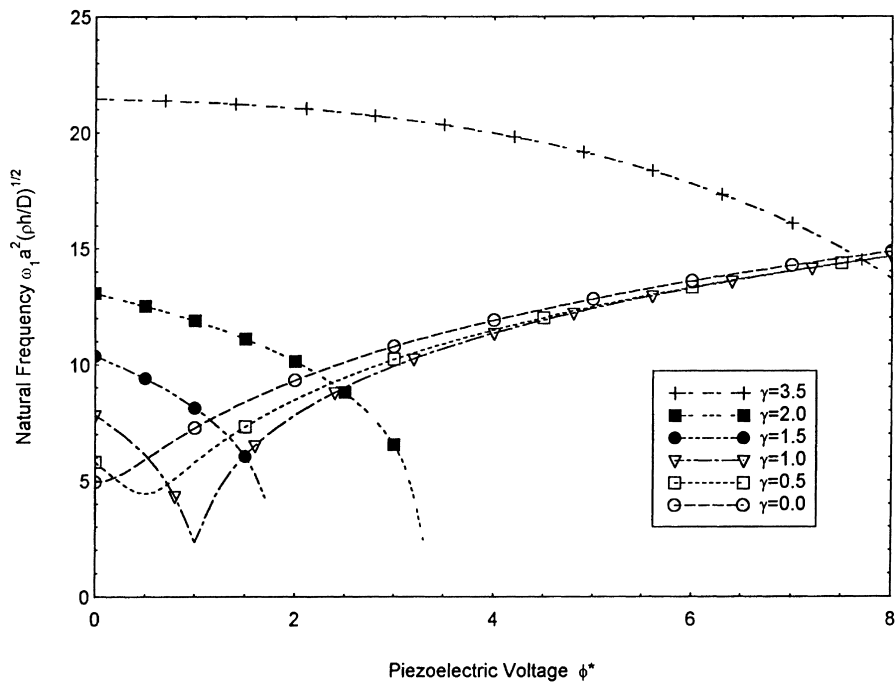


Fig. 4. Change of the first natural frequency $\omega_1 a^2 (\rho h / D)^{1/2}$ vs control voltage $\phi^* = 6(1 - \mu^2)[3(1 - \mu^2)]^{1/2} e_{31}(h + h_p)\phi_3^u / Yh^4$. $\mu = 0.3$.

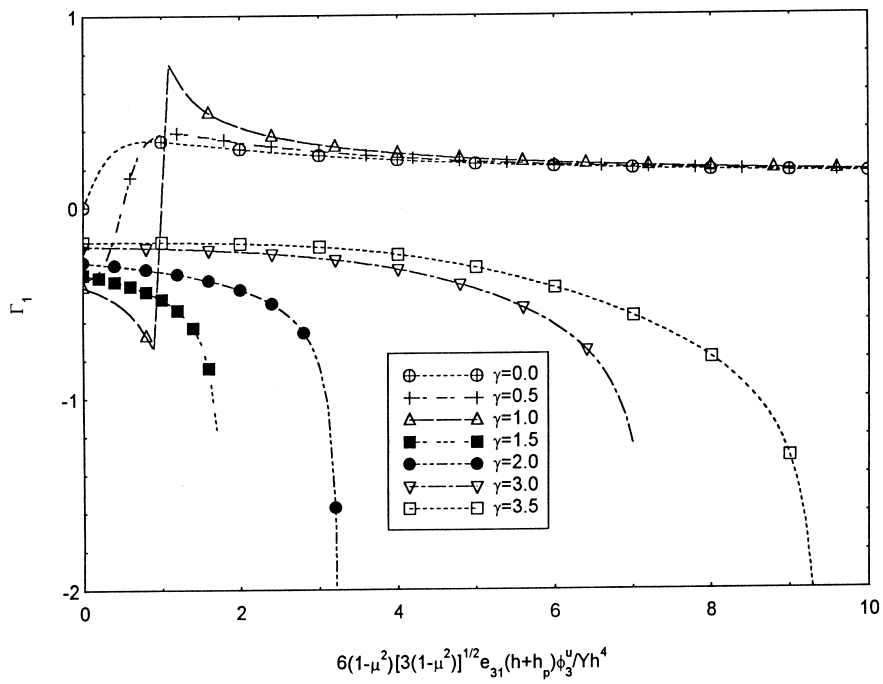


Fig. 5. Nonlinear coefficient Γ_1 of the first mode vs control voltage $\phi^* = 6(1 - \mu^2)[3(1 - \mu^2)]^{1/2}e_{31}(h + h_p)\phi_3^u/Yh^4$. $\mu = 0.3$.

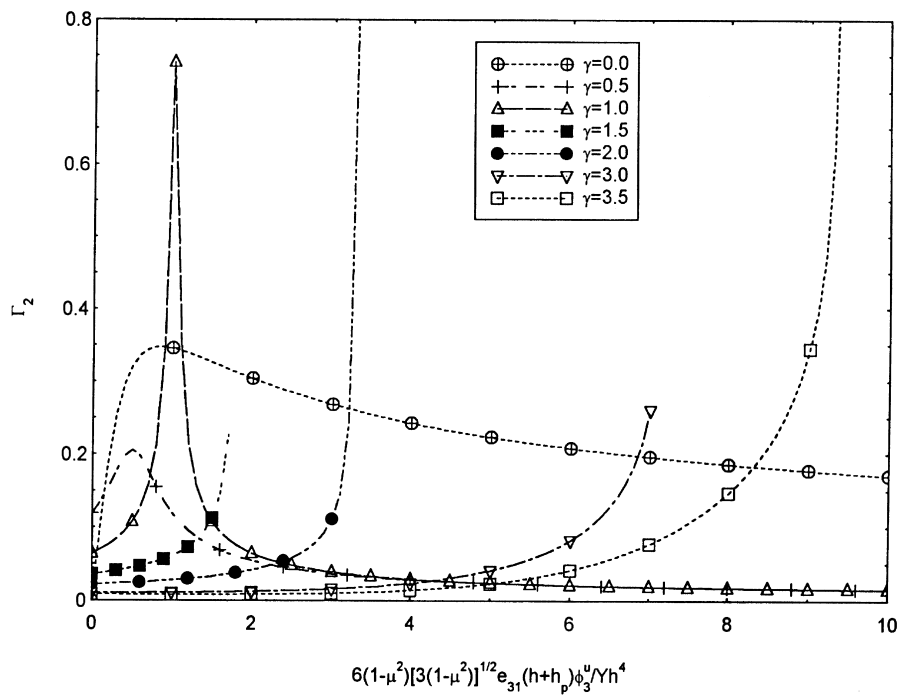


Fig. 6. Nonlinear coefficient Γ_2 of the first mode vs control voltage $\phi^* = 6(1 - \mu^2)[3(1 - \mu^2)]^{1/2}e_{31}(h + h_p)\phi_3^u/Yh^4$. $\mu = 0.3$.

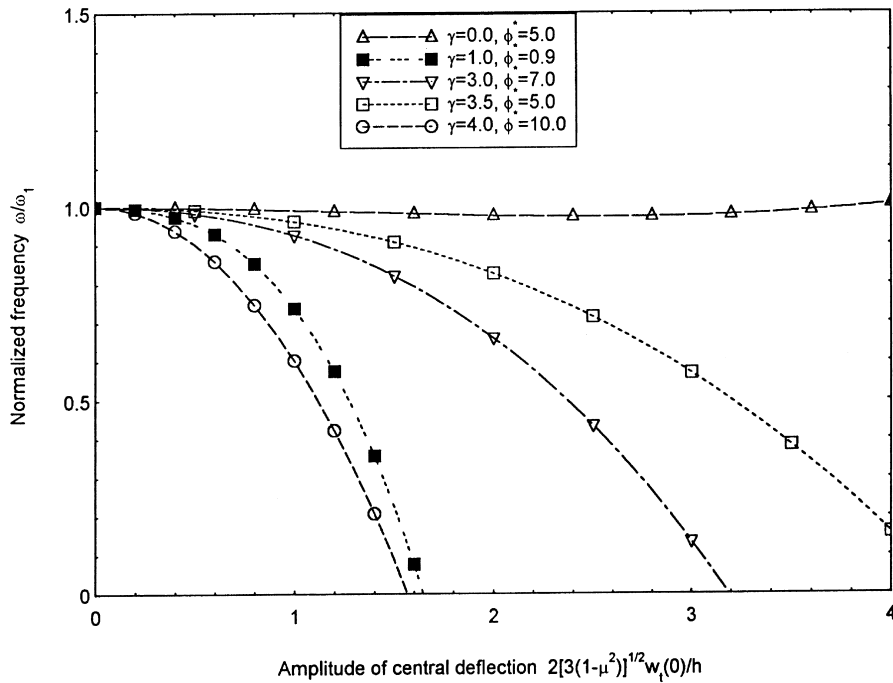


Fig. 7. Natural frequency ω/ω_1 and amplitude $2[3(1-\mu^2)]^{1/2}w_1(0)/h$ relations for different control voltage $\phi^* = 6(1-\mu^2)[3(1-\mu^2)]^{1/2}e_{31}(h+h_p)\phi_3^0/Yh^4$ and for different geometrical parameter $\gamma = [3(1-\mu^2)]^{1/2}f/h$. $\mu = 0.3$.

$$p^* = \frac{\omega}{\omega_n} = 1 + \frac{1}{24}(9\Gamma_2 - 10\Gamma_1^2)\hat{A}^2 + \frac{5}{8}\Gamma_1\Gamma_2\hat{A}^3 + \dots \tag{63}$$

where ω is the frequency of nonlinear vibration; \hat{A} denotes the amplitude of dimensionless deflection. When the test function $U^*(x)$ is normalized by the form

$$U^*(x)|_{x=0} = 1 \tag{64}$$

\hat{A} is the amplitude of the dimensionless deflection at the center point of the shell in the vicinity of the nonlinear static state. That is

$$\hat{A} = 2[3(1-\mu^2)]^{1/2}\frac{w_1}{h}\Big|_{r=0} \tag{65}$$

and the modal function (assumed eigenfunction) $U^*(x)$ is taken from the (exact) series-type eigenfunctions $R_l(x)$ in eqn (45a) for small vibration.

6. Numerical results and analysis (case study)

According to the analysis, a computer program was performed by computer for finding solutions of the nonlinear piezoelectric circular shallow spherical shells. Due to the vibration of first mode is more prominent than others in practice, here, we restrict our attention to finding the dynamic characteristic of the first mode of vibration of the piezoelectric shells. Fig. 2 shows the characteristic curves of central

deflection $[3(1 - \mu^2)]^{1/2} w_0(0)/h$ with applied voltage $\phi^* = 6(1 - \mu^2)[3(1 - \mu^2)]^{1/2} e_{31}(h + h_p)\phi_3^u/Yh^4$. The natural frequency of small free vibration with vibration mode shown in Fig. 3 in the vicinity of the nonlinear deformation configuration is plotted in Fig. 4. Figures 5 and 6 indicate the varying of coefficients Γ_1 and Γ_2 with applied piezoelectric voltage, respectively. From Figs. 2 and 4, it can be found that the jumping or snapping phenomenon of the piezoelectric shallow spherical shell may occur for the cases of the shell with $\gamma \geq 1$ when the applied voltage approaches a critical value. Under this value, the natural frequency of the shell decreases as applied voltage increases. It may capture the critical value of applied voltage according to the condition that the natural frequency is approached to zero. Corresponding to this snapping phenomenon, the coefficients Γ_1 and Γ_2 related to nonlinear terms in eqn (58) change rapidly when the applied voltage approaches the critical value. However, they change a little for the case $\gamma < 1$ in which there is no jumping phenomenon. Fig. 7 shows the effect of large deflection on the natural frequency of the piezoelectric shells. When the applied voltage is under the critical value for the case $\gamma \geq 1$, it is known that the vibrating frequency decreases as the amplitude increases. As the frequency of vibration approaches zero, the nonlinear free vibration may lose stability.

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References

- Hubbard, J.E., Burke, S.E., 1992. Distributed transducer design for intelligent structural components. In: Tzou, H.S., Anderson, G.L. (Eds.), *Intelligent Structural Systems*. Kluwer Academic Publishers, Dordrecht, Boston, London, pp. 305–324.
- Hwang, W.S., Park, H.C., 1993. Finite element modelling of piezoelectric sensors and actuators. *AIAA Journal* 31 (5), 930–937.
- Lalande, F., Chaudhry, Z., Roger, C.A., 1993. A simplified geometrically nonlinear approach to the analysis of the Moonier actuator. *Adaptive Structures and Material Systems, AD-Vol. 35. ASME WAM, New Orleans, LA, 28 November–3 December, 1993*, pp. 149–155.
- Librescu, L., 1987. Refined geometrically nonlinear theories of anisotropic laminated shells. *Quarterly of Applied Mathematics* XLV (1), 1–22.
- Nayfeh, A.H., Mook, D.T., 1979. *Nonlinear Oscillations*. John Wiley and Sons, New York.
- Pai, P.F., Nayfeh, A.H., Oh, K., Mook, D.T., 1993. A refined nonlinear model of piezoelectric plate with integrated piezoelectric actuators and sensors. *Int. J. Solids and Structures* 30, 1603–1630.
- Plump, J.M., Hubbard, J.E., Baily, T., 1987. Nonlinear control of a distributed system: simulation and experimental results. *ASME J. of Dynamic Systems, Measurement and Control* 109, 133–139.
- Sreeram, P.N., Salvady, G., Naganathan, N.G., 1993. Hysteresis prediction for a piezoceramic material system. *Adaptive Structures and Material Systems, AD-Vol. 35. ASME WAM, New Orleans, LA, 28 November–3 December, 1993*, pp. 35–42.
- Tzou, H.S., 1993. *Piezoelectric Shells (Distributed Structural Systems)*. Kluwer Academic February 1993.
- Tzou, H.S., Anderson, G.L. (Eds.), 1992. *Intelligent Structural Systems*. Kluwer Academic, Dordrecht, Boston, London August 1992.
- Tzou, H.S., Tsing, C.I., 1991. Distributed model identification and vibration control of continua: piezoelectric finite element formulation and analysis. *ASME Journal of Dynamics, Measurement and Control* 113, 500–505.
- Tzou, H.S., Zhong, J.P., 1993. Electromechanics and vibrations of piezoelectric shell distributed systems: theory and applications. *ASME J. Dynamic Systems, Measurements and Control* 115 (3), 506–517.
- Tzou, H.S., Zhou, Y.H., 1995. Dynamics and controls of piezoelectric circular plates with geometrical nonlinearity. *Journal of Sound and Vibration* 188, 189–207.
- Yu, Y.Y., 1993. Some recent advances in linear and nonlinear dynamical modeling of elastic and piezoelectric plates. *Adaptive Structures and Material Systems, AD-Vol. 35. ASME WAM, New Orleans, LA, 28 November–3 December, 1993*, pp. 185–195.

- Zheng, X.J., 1990. Theory of Large Deflection of Circular Plates and its Applications. Publishing House of Science and Technology of Jiling, Changchun, China (in Chinese).
- Zheng, X.J., Zhou, Y.H., 1990. Analytical formula of solutions of geometrically nonlinear equations of axisymmetric plates and shells. *Acta Mechanica Sinica* 6, 69–80.
- Zhou, Y.H., 1989. Flexural Bending, Vibration and Stability of Nonlinear Structures under Static Loads. Ph.D. dissertation, Lanzhou University.
- Zhou, Y.H., Yeh, K.Y., Rimrott, F.P.J., 1994. Theory of vibration diaphragm type pressure sensor. *AIAA Journal* 32, 633–638.